the **Availability Digest**

Calculating Availability – Repair Strategies

November 2006

Review

In our previous article on Calculating Availability – Redundant Systems, we demonstrated that the availability of a multinode active/active system is

$$A = 1 - f(1-a)^{s+1} \tag{1a}$$

and its probability of failure is

$$F = 1 - A = f(1-a)^{s+1} \tag{1b}$$

where

- *A* is the probability that the system will be up (the system's availability).
- *F* is the probability that the system will be down.
- *a* is the probability that a node will be up (a node's availability).
- *s* is the number of spare nodes in the system.
- *f* is the number of failure modes. It is the number of ways that all of the spares plus one additional node can fail.

If there are *s* spare nodes in the system, the system will survive the failure of any combination of *s* nodes. Therefore, *s*+1 nodes must fail in order for the system to be unavailable. However, only one of these failed nodes needs to be repaired in order to return the system to service.[1]

You can bypass the math and go directly to the Summary if you wish.

The Repair Strategy

Relations (1a) and (1b) were casual in terms of how the repair of multiple failed nodes was handled. As it turns out, there are two repair strategies that are commonly used; and which one is used has a marked impact on the availability equations through its impact on the number of failure modes, *f*. These repair strategies are:

[1] The above analysis and the analyses which follow are taken from the books entitled *Breaking the Availability Barrier: Survivable Systems for Enterprise Computing*, and *Breaking the Availability Barrier: Achieving Century Uptimes with Active/Active Systems*, by Dr. Bill Highleyman, Paul J. Holenstein, and Dr. Bruce Holenstein.

parallel repair, via which service technicians are dispatched to all failed nodes and work on them simultaneously to be repaired.

sequential repair, via which there is only one service technician who repairs the nodes one at a time.

Clearly, we would expect that parallel repair would be advantageous over sequential repair since we are applying more repair effort to the system, but to what extent is this an advantage? We analyze these two cases and come up with what may be some surprising results.

Parallel Repair

Let us take the simple case of a singly-spared system (that is, s = 1). In our previous article, we noted that the maximum number of failure modes for this case is

$$f = \frac{n(n-1)}{2}$$

where n is the number of nodes in the system.

This is explainable as follows. The system will fail if two nodes fail simultaneously. There are n ways that one node of the n operable nodes can fail. Given that one node has failed, leaving just n-1 operable nodes in the system, there are n-1 ways that a second node can fail. Therefore, there are $n(n$-1) ways that two nodes in the system can fail, thus taking it down. However, this argument has counted each node pair twice. For instance, a node 4 failure followed by a node 3 failure has been counted as well as a node 3 failure followed by a node 4 failure. Therefore, the count is high by a factor of 2, and we correct for this by dividing the result by 2.

As we consider this argument in the light of repair strategies, we realize that we have assumed that the order of node failure is not important. No matter which node fails first, the node that is repaired first is the one that returns the system to service. This implies that the nodes are being repaired simultaneously, since either one could be repaired before the other. This is parallel repair.

Thus, for the parallel repair strategy:

$$f = \frac{n(n-1)}{2} \quad \text{for a singly-spared system with parallel repair} \qquad (2)$$

Sequential Repair

If there is only one service technician, the order in which the nodes fail is important because the first node that failed will be the first to be returned to service. Therefore, the maximum number of failure modes when a sequential repair strategy is used is

$$f = n(n-1) \quad \text{for a singly-spared system with sequential repair} \qquad (3)$$

There are twice as many failure modes when sequential repair is used in a singly-spared system than when parallel repair is used. Therefore, the probability of a system failure, which is proportional to f, is twice as large for sequential repair as it is for parallel repair.

The Impact of Repair Strategy on System Probability of Failure

The above analysis has concentrated on singly-spared systems. But what if there is more than one spare?

The difference between the number of failure modes for parallel or sequential repair is the difference in the number of ways that any given set of *s*+1 nodes can be chosen for failure. There are *s*+1 ways that the first node to fail can be chosen, *s* ways for the second node, *s*-1 ways for the third node, and so on until the last node, for which there is only one choice. Therefore, the number of different ways that the same set of *s*+1 nodes can be chosen is (*s*+1)!. This is the parallel repair advantage and leads to the following rule:

> *Parallel Repair Failure Probability Advantage Rule*: If there are *s* spares configured for the system, parallel repair will reduce system failure probability by a factor of (*s*+1)! as compared to sequential repair.

For instance, if there is one spare node, parallel repair will have a 2:1 advantage over sequential repair. If there are two spares, parallel repair will have a 6:1 advantage.

The Impact of Repair Strategy on System MTR

The amount of time that a system is down, its mean time to repair, or MTR, is the time that it takes to repair the first node so that it can be returned to service.

For sequential repair, the mean time to repair the system, MTR, is, in fact, the nodal repair time mtr (we will use lower case *mtr* and *mtbf* for a node and upper case *MTR* and *MTBF* for the system). Thus,

$$\text{MTR} = \text{mtr} \qquad \text{for sequential repair of a singly-spared system} \qquad (4)$$

For parallel repair, we have to be a little more careful. On the average, a service technician will make one repair during each time interval of mtr, the node's mean time to repair. Remember that this is an average time to repair – some repairs will take much longer, and some will be much faster.

If there are two service technicians working independently on two failed nodes, in the time mtr there will be on the average two repairs. Thus, there will be an average of one repair during every mtr/2 time interval. For instance, if nodal mtr is four hours, and there are two downed nodes that are being worked on by two different service technicians, on the average there will be two repairs done in the four-hour period, or one repair on the average every two hours. Therefore, in this case, it will take on the average two hours to repair the first node and return the node to service.

Thus, for parallel repairs, the average system MTR for a singly-spared system is one-half the nodal mtr:

$$\text{MTR} = \text{mtr}/2 \qquad \text{for parallel repair of a singly-spared system} \qquad (5)$$

If the active/active network has more than one spare, we can derive similar rules by following the reasoning above. Sequential repair is independent of the number of nodes in the system, and its MTR relation stays the same:

$$\text{MTR} = \text{mtr} \qquad \text{for sequential repair of a multiple-spared system} \qquad (6)$$

If a downed system with *s* spare nodes is being repaired, there are now *s*+1 service technicians working on the same number of nodes. In a time interval mtr, there will be *s*+1 repair completions; and the average repair time for one node will be mtr/(*s*+1). Thus,

$$\text{MTR} = \text{mtr}/(s+1) \qquad \text{for parallel repair of a multiple-spared system} \qquad (7)$$

This leads to the following rule:

> *Parallel Repair MTR Advantage Rule:* If there are *s* spares configured for the system, parallel repair will reduce system MTR by a factor of (*s*+1) as compared to sequential repair.

The Impact of Repair Strategy on System MTBF

Since by our above rules, parallel repair reduces system failure probability by a factor of (*s*+1)! and system MTR by a factor of (*s*+1), then system MTBF must be increased by a factor of *s*!. This leads to the following corollary:

> *Parallel Repair MTBF Advantage Rule:* If there are *s* spares configured for the system, parallel repair will increase system MTBF by a factor of *s*! as compared to sequential repair.

The Importance of Nodal Repair Time

The Single-Spared Nodal Repair Time Rule

Over and above the repair strategy that is used when a multiple node failure takes down a system, as discussed above, there is significant leverage for reducing system failure probability by simply reducing the repair time of a node.

Let us return to the basic availability equation which states that the probability of failure of a system is the proportion of the time that it is down:

$$F = 1 - A = \frac{\text{MTR}}{\text{MTBF} + \text{MTR}} \approx \frac{\text{MTR}}{\text{MTBF}} \qquad (8)$$

where

F	is the probability of failure of the system.
A	is the availability of the system.
MTR	is the average amount of time that the system is down following a failure (the mean time to repair).
MTBF	is the average time that the system is up following a repair (the mean time before failure).

That is, the probability of failure of the system is the time that it is down (MTR) as compared to the total time (MTBF+MTR, since the system is either down or up). The approximation is good if MTR << MTBF, which is certainly true in the cases in which we are interested.

Let us take one more step before continuing. Based on the basic availability equation, F = 1-A ≈ MTR/MTBF, we note that the failure probability of a node, (1-*a*), is approximately mtr/mtbf, where mtr is the average repair time for a node (its mean time to repair), and where mtbf is a node's average uptime (its mean time before failure).

Thus, using Equations (1b) and (8), our basic availability equation can be rewritten as

$$F = 1 - A \approx \frac{MTR}{MTBF} = f\left(\frac{mtr}{mtbf}\right)^2 \qquad (9)$$

For instance, if we can reduce nodal repair time by a factor of two, we can decrease the probability of system failure by a factor of four (it is proportional to mtr^2).

> *Single-Spared Nodal Repair Time Rule*: In a system with one spare, reducing the nodal repair time by a factor of *k* will reduce the system probability failure by a factor of k^2.

The Multi-Spared Nodal Repair Time Rule

For multiple spares, the system failure equation expressed above as Equation (9) becomes

$$F = 1 - A \approx \frac{MTR}{MTBF} = f\left(\frac{mtr}{mtbf}\right)^{s+1} \qquad (10)$$

The system failure probability is a function of $(mtr)^{s+1}$. If we can reduce the nodal repair time by a factor of *k*, the system probability of failure has been decreased by a factor of $k^{s+1.}$ For instance, in a dually-spared system, if the nodal repair time can be reduced by 2, the system probability of failure is reduced by a factor of eight. This leads to the following rule:

> *Multi-Spared Nodal Repair Time Rule*: In a system with *s* spares, reducing the nodal repair time by a factor of *k* will reduce the system probability failure by a factor of k^{s+1}.

Note that for a single-spared system ($s = 1$), this reduces to our single-spared repair rule given above.

Impact of Nodal Repair Time on System MTBF

We know the reduction in the system probability of failure due to reduced nodal repair time from the above equations. We can determine the impact of reduced nodal repair time on system MTR from Equations (6) and (7). Knowing these parameters, we can determine the increase in system MTBF due to a reduced nodal repair time from a rearrangement of Equation (10):

$$MTBF = \frac{MTR}{F} \qquad (11)$$

Summary

We have analyzed the difference between parallel repair and sequential repair and have seen that parallel repair provides significantly improved availability as expressed by our Repair Strategy Rules:

> *Parallel Repair Failure Probability Advantage Rule*: If there are *s* spares configured for the system, parallel repair will reduce system failover probability by a factor of (*s*+1)! as compared to sequential repair.

> *Parallel Repair MTR Advantage Rule:* If there are *s* spares configured for the system, parallel repair will reduce system MTR by a factor of (*s*+1) as compared to sequential repair.

Parallel Repair MTBF Advantage Rule: If there are *s* spares configured for the system, parallel repair will increase system MTBF by a factor of *s*! as compared to sequential repair.

If one spare node is provided, parallel repair reduces the probability of failure by a factor of two and reduces system MTR by a factor of two. If two spares are provided, failure probability is reduced by a factor of six, system MTR is reduced by a factor of three, and system MTBF is increased by a factor of two.

Clearly, if one is going to go to the expense and effort of an active/active system, one must ensure that the service team is large enough and distributed widely enough to effectively provide parallel repair.

On a similar course, reducing nodal repair time can also have significant impact on system failure probability, as expressed by our Nodal Repair Time Rules:

Single-Spared Nodal Repair Time Rule: In a system with one spare, reducing the nodal repair time by a factor of k will reduce the system probability failure by a factor of k^2.

Multi-Spared Nodal Repair Time Rule: In a system with *s* spares, reducing the nodal repair time by a factor of k will reduce the system probability failure by a factor of k^{s+1}.

For instance, in a system with one spare, reducing nodal repair time by a factor of two will reduce the probability of system failure by a factor of four. If two spares are provided, the system probability failure will be reduced by a factor of eight.