

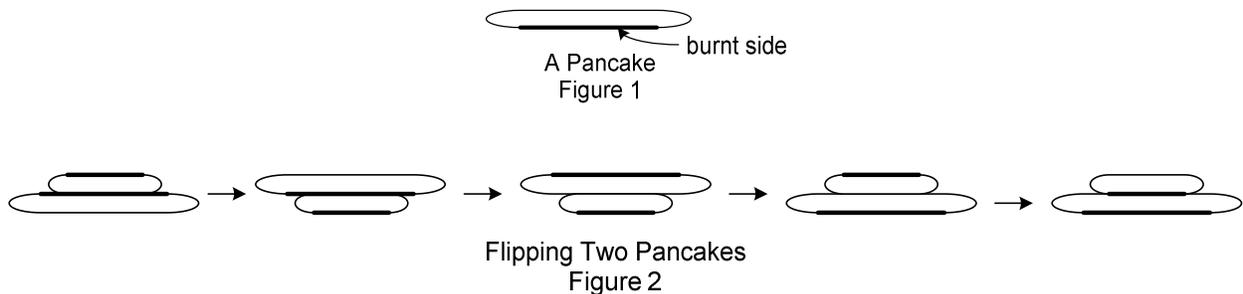
## The Pancake Problem

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Jacob Goodman was a mathematician at the City College of New York. One day, at home folding towels for his wife, Goodman decided to restack the towels in the order of size, smallest folded towel on top. The problem was, there was no room for a second stack. He was forced to flip over a few towels at a time until they were ordered. 

A curious problem crossed his mind. How many flips would it take in the worst case to order the towels according to size? Thus, the so-called pancake sorting problem was born.

Two pancakes would take at most one flip. But if the pancakes were burnt on one side as shown in Figure 1, and the resulting stack must have all pancakes with their burnt sides down, then it could take four flips, as shown in Figure 2.



The problem is simple to pose, yet tough to solve. No one is close to developing a general formula that predicts the number of flips required for any given number of pancakes.

Interestingly, the best approach for almost 30 years was written by Bill Gates in his only article of scientific research. His co-authored paper proved an upper limit to reorder  $n$  unburnt pancakes to be  $5(n+1)/3$ . Therefore, if you have 100 unburnt pancakes, you will always be able to order them correctly with 169 or fewer flips.

As the pile of pancakes grows, the problem becomes increasingly difficult as there are more possible starting positions. For 17 pancakes, it could take 19 flips. For 18 pancakes, it could take 20 flips. For 19 pancakes, it could take 22 flips. Nobody knows the number of flips required to reorder 20 pancakes. The determination of this bound hasn't been possible even with the help of today's most powerful computers.

30 years later, the minimum number of flips to sort  $n$  unburnt pancakes has been shown to lie between  $15n/14$  and  $18n/11$  (approximately  $1.07n$  and  $1.64n$ ).

Bill Gates and Christos Papadimitriou introduced the burnt pancake problem. They showed that  $5(n+1)/3$  flips always suffices. For burnt pancakes with the burnt side down, the number of flips is bounded by  $3n/2$  and  $2n-2$ . Thus, for 100 burnt pancakes, the number of flips required is between 150 and 198.

## Acknowledgements

Information for this article was taken from the following sources:

Flipping pancakes with mathematics, *The Guardian*; November 14, 2014.

Bill Gates and the Pancake Problem, *Science*; July 10, 2018.

The Pancake Problems, *University of Illinois Math Faculty*; undated.

Pancake sorting, *Wikipedia*.